

Fast Spectrum Sensing using a Large Number of Receive Antennas

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Abstract—In this paper, we propose a novel spectrum sensing method to detect the presence of a primary user in a cognitive radio network when the secondary user is employing a large number of receive antennas. The new method is a suboptimal approach based on the generalized likelihood ratio test (GLRT), but unlike the previous GLRT-based methods which rely on large number of received samples obtained over time, we take advantage of the large number of samples available at the receive antennas. By exploiting the properties of the samples obtained at the receive antennas, effective spectrum sensing is achieved using mainly the spatial samples and therefore the sensing time is significantly reduced. With the suitable channel models and prior knowledge about the signal structure, the simulation results show that the proposed method offers performance comparable to well-known methods but with much smaller sensing time.

Index Terms—spectrum sensing, cognitive radio

I. INTRODUCTION

Spectrum sensing is an indispensable component of cognitive radio (CR), which is commonly regarded as a form of communication in which devices may intelligently detect and switch to vacant channels. The main task of spectrum sensing is to let the secondary users, who seek to opportunistically use the channels unoccupied by their legal owners, detect whether those channels of interest are idle. Thus, the accuracy of detection and the time required for making credible decisions become the essential metrics of the spectrum sensing (SS) algorithms. In the past, numerous SS algorithms have been proposed and there exist a number of good surveys of these algorithms in the literature [1] [2]. Typically, SS algorithms fall under categories such as energy detection, matched filtering, cyclostationary-based detection, eigenvalue-based and GLRT-based detection [3] [4]. These algorithms vary in terms of their performances, complexities, the signal properties assumptions and knowledge of the signal and noise parameters. SS techniques are also employed in multi-antenna systems [7] [8].

Among the SS algorithms, the GLRT-based and eigenvalue-based detectors are examples of constant false-alarm rate detectors as their false alarm probabilities are independent of the noise parameters. By exploiting the signal's second-order statistics, the detector's threshold can be fixed in spite of the noise level and thus they could potentially outperform the energy detector when the knowledge of the noise is absent. In this paper, our goal is to exploit the known channel properties of receive antenna array, while the noise parameter

is unknown. Thus, the GLRT-based approach is a natural fit in our treatment of SS.

Furthermore, we focus on SS in a SIMO (single-input-multiple-output) scenario in which the secondary user is equipped with a large receive antenna array. Currently, spectrum sensing in massive MIMO system is still in its infancy. In [9] [10], random matrix models have been employed for the computation of eigenvalues which are essential in determining the test statistics. Authors in [10] demonstrated the convergence of the eigenvalue distribution for very large random matrix through simulations and suggested its potential usage in SS. However, to the best of our knowledge, there are no direct results on the derivation of test statistics for SS in massive MIMO system. While eigenvalue-based or GLRT-based approaches such as those in [7] [8] could be applied in massive MIMO system, the complexity involved in the calculation of eigenvalues grows tremendously as the number of antennas increases. Due to this reason, we propose a simple approach based on the GLRT method for spectrum sensing with a large number of receive antennas. In our method, by exploiting the channel properties, the large number of spatial samples are utilized in a similar fashion as the extra time domain samples to improve the eigenvalue-based test statistic whose computation involves only small covariance matrices. As a result, given a few time domain samples, our method may yield comparable performance as other GLRT-based methods using a large number of time domain samples, yet with similar level of complexity. In other words, the advantage of using large receive antenna array translates into the reduction of the time domain samples required for accurate spectrum sensing.

The rest of the paper is organized as follows. Section II describes the system model. Section III presents the GLRT-based detection and our proposed method. Section IV contains simulation results and discussion. Finally, Section V concludes the paper.

II. SYSTEM MODEL

We consider a quasi-static flat fading channel with the channel coefficients given by $h_m, m \in \{1, \dots, M\}$ where h_m denotes the channel from the primary user to the m -th antenna at the secondary user's receiver. In spectrum sensing, the secondary user's receiver needs to choose between the following hypothesis

$$\mathcal{H}_0 : \mathbf{x}(n) = \mathbf{w}(n); \quad (1)$$

$$\mathcal{H}_1 : \mathbf{x}(n) = \mathbf{h}d(n) + \mathbf{w}(n); \quad (2)$$

where $\mathbf{h} = [h_1, h_2, \dots, h_M]^T$, $\mathbf{x}(n) = [x_1(n), x_2(n), \dots, x_M(n)]^T$, $\mathbf{w}(n) = [w_1(n), w_2(n), \dots, w_M(n)]^T$, $w_m(n)$ denotes the i.i.d. Gaussian noise sample with variance σ^2 , $d(n)$ is the i.i.d. BPSK-modulated data and $n \in \{0, \dots, N-1\}$ denotes the time index. In this paper, M is supposed to be very large whereas N is assumed to be small ($N \leq 2$) for fast sensing. As for the channel coefficients, we consider two models: (1) phased-array model, and (2) Rayleigh fading model. For the phased-array model, the channel vector is given as follows:

$$\mathbf{h} = \mathbf{a}(\theta)h \quad (3)$$

where $\mathbf{a}(\theta) = [e^{j2\pi D \sin \theta}, \dots, e^{j2\pi(M-1)D \sin \theta}]^T$ is the array response vector (D is the element spacing in wavelengths) with the arrival angle $\theta \in [0, 2\pi)$ and h is a complex Gaussian random variable with unit variance ($\sigma_h^2 = 1$). And for the Rayleigh fading model, the channel coefficients, $h_m, m \in \{1, \dots, M\}$, are i.i.d. complex Gaussian random variables with unit variance.

III. GLRT-BASED DETECTION AND THE PROPOSED METHOD

GLRT-based spectrum sensing is essentially a composite hypothesis test in which the binary hypothesis test, deciding between $\mathcal{H}_1$ and $\mathcal{H}_0$, is carried out in the presence of unknown parameters. This method first obtains the maximum likelihood (ML) estimate of the parameters under $\mathcal{H}_1$ and $\mathcal{H}_0$:

$$\hat{\Theta}_0 = \arg \max_{\Theta_0} p(\mathbf{x}|\mathcal{H}_0, \Theta_0) \quad (4)$$

$$\hat{\Theta}_1 = \arg \max_{\Theta_1} p(\mathbf{x}|\mathcal{H}_1, \Theta_1) \quad (5)$$

and then followed by forming the GLRT-statistic:

$$L_G(\mathbf{x}) = \frac{p(\mathbf{x}|\hat{\Theta}_1, \mathcal{H}_1)}{p(\mathbf{x}|\hat{\Theta}_0, \mathcal{H}_0)} \quad (6)$$

The test decides $\mathcal{H}_1$ if $L_G(\mathbf{x}) > \gamma$ where γ is the threshold. The GLRT-based method has been employed in [4] [5] [6]. All the GLRT-based works assume that a number of signal samples are available in order to estimate the covariance matrix,

$$\hat{\mathbf{R}}_x = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}(n)\mathbf{x}^H(n) \quad (7)$$

In [4], the GLRT-based method treats the $\mathbf{R}_x$ and the noise variance as unknown parameters (that is, $\Theta_1 = [\mathbf{R}_x, \sigma^2]$, $\Theta_0 = [\sigma^2]$). Using the covariance matrix estimation (7), the test statistics is shown to be a function of the eigenvalues, denoted as $\hat{\lambda}_m, m \in \{1, \dots, M\}$, of $\hat{\mathbf{R}}_x$:

$$T_{GLR} = \frac{1/M \sum_m \hat{\lambda}_m}{(\prod_m \hat{\lambda}_m)^{1/M}} \quad (8)$$

Depending on whether the knowledge of the rank of the signal subspace or the noise variance is available, the above test statistics can be modified, leading to variations

of GLRT-based method, which are also functions of the eigenvalues of $\hat{\mathbf{R}}_x$ [4]. Besides GLRT-based methods, there are other existing SS algorithms using the eigenvalues of $\hat{\mathbf{R}}_x$, which again requires the availability of a large number of signal samples in the time domain in order to obtain accurate estimation of the covariance matrix.

A. Multiantenna-assisted GLRT-based method (M-GLRT)

With regard to the spectrum sensing for large number of receive antennas, we are aware of several existing GLRT-based methods for multiple receive antennas. For comparison purposes, the multiantenna-assisted GLRT-based method (M-GLRT) proposed in [7] is explained here. Compared with the GLRT method [4] described above, this method exploits the inherent signal structure and thus leads to improved performance. Its test statistic can be written as,

$$T_{M-GLR} = \frac{\max_{\mathbf{h}, \sigma^2} p(\mathbf{X}|\mathcal{H}_1, \mathbf{h}, \sigma^2)}{\max_{\sigma^2} p(\mathbf{X}|\mathcal{H}_0, \sigma^2)} \quad (9)$$

where $\mathbf{X} = [\mathbf{x}(0), \mathbf{x}(1), \dots, \mathbf{x}(N-1)]$ and

$$p(\mathbf{X}|\mathcal{H}_0, \sigma^2) = \prod_{n=0}^{N-1} \frac{1}{\pi^M \sigma^{2M}} \exp \left[-\frac{\mathbf{x}^H(n)\mathbf{x}(n)}{\sigma^2} \right] \quad (10)$$

$$p(\mathbf{X}|\mathcal{H}_1, \mathbf{h}, \sigma^2) = \prod_{n=0}^{N-1} \frac{1}{\pi^M |\mathbf{h}\mathbf{h}^H + \sigma^2 \mathbf{I}|} \times \exp \left[-\frac{\mathbf{x}^H(n)(\mathbf{h}\mathbf{h}^H + \sigma^2 \mathbf{I})^{-1}\mathbf{x}(n)}{\sigma^2} \right] \quad (11)$$

The ML estimates of unknown $\mathbf{h}$ and σ^2 are found to be [7],

$$\hat{\mathbf{h}}_{ML} = \left(\hat{\lambda}_1 - \frac{1}{M-1} \sum_{m=2}^M \hat{\lambda}_m \right)^{1/2} \mathbf{U}_x[:, 1] \quad (12)$$

$$\hat{\sigma}_{\mathcal{H}_1}^2 = \frac{1}{M-1} \sum_{m=2}^M \hat{\lambda}_m \quad (13)$$

$$\hat{\sigma}_{\mathcal{H}_0}^2 = \frac{1}{M} \sum_{m=1}^M \hat{\lambda}_m \quad (14)$$

where the eigenvalues, $\hat{\lambda}_1, \dots, \hat{\lambda}_M$, of $\hat{\mathbf{R}}_x$ are arranged in descending order, and $\mathbf{U}_x[:, 1]$ denotes the eigenvector corresponding to the largest eigenvalue $\hat{\lambda}_1$. With the ML estimates, the test statistic can be expressed as,

$$T_{M-GLR} = \frac{\hat{\lambda}_1}{\sum_{m=1}^M \hat{\lambda}_m} \frac{H_0}{H_1} \gamma_{GLR} \quad (15)$$

The main computational burden of this method lies in the calculation of eigenvalues. For massive MIMO system where M is very large, this method may become impractical due to its high complexity.

B. Proposed method

Our method exploits the signal structure in a similar fashion as the M-GLRT method described above does. To reduce complexity, the proposed approach partitions the spatial samples

of length M into subgroups, each of which has length equal to P . Thus, $\mathbf{x}(n)$ can be expressed as

$$\mathbf{x}(n) = [\mathbf{x}_0^T(n), \mathbf{x}_1^T(n), \dots, \mathbf{x}_g^T(n), \dots, \mathbf{x}_{G-1}^T(n)]^T \quad (16)$$

where $\mathbf{x}_g(n) = [x_{gP+1}(n), x_{gP+2}(n), \dots, x_{(g+1)P}(n)]^T$ and $G = M/P$. In order to find the likelihood functions, it is noticed that conditioned on $\mathbf{h}$ and the input $d(n)$, these subgroups of received samples are P -dimensional complex Gaussian vectors and they are independent of one another, since their conditional covariance is zero, i.e., $\text{cov}(\mathbf{x}_a(n), \mathbf{x}_b(n)) | \mathbf{h}, d(n) = 0, \forall a \neq b$. As such, we can formulate the likelihood functions under $\mathcal{H}_1$ and $\mathcal{H}_0$ as follows,

$$p(\mathbf{X} | \mathcal{H}_0, \sigma^2) = \prod_{g=0}^{G-1} p(\mathbf{X}_g | \mathcal{H}_0, \sigma^2) \quad (17)$$

$$p(\mathbf{X} | \mathcal{H}_1, \mathbf{h}, \sigma^2, d(n)) = \prod_{g=0}^{G-1} p(\mathbf{X}_g | \mathcal{H}_1, \mathbf{h}_g, \sigma^2, d(n)) \quad (18)$$

where $\mathbf{h}_g = [h_{gP+1}, h_{gP+2}, \dots, h_{(g+1)P}]^T$ and $\mathbf{X}_g = [\mathbf{x}_g(0) \dots \mathbf{x}_g(N-1)]$. To simplify the calculation of the test statistic, we now make an approximation: the M-GLRT test statistic can be employed for each subgroup, that is

$$\frac{p(\mathbf{X}_g | \mathcal{H}_1, \mathbf{h}_g, \sigma^2, d(n))}{p(\mathbf{X}_g | \mathcal{H}_0, \sigma^2)} \rightarrow \frac{p(\mathbf{X}_g | \mathcal{H}_1, \mathbf{h}_g, \sigma_g^2)}{p(\mathbf{X}_g | \mathcal{H}_0, \sigma_g^2)} \quad (19)$$

The above approximation relaxes the requirement of estimating the $d(n)$ in the GLRT statistic calculation by assuming that it is a random quantity and no longer a parameter. Note that $d(n)$ can be treated as a binary random variable when N is large. As such, we may employ the M-GLRT test statistic for each subgroup and the final test statistic is the product of all subgroup's statistic. The proposed test statistic can be expressed as,

$$\begin{aligned} T_{\text{proposed}} &= \frac{\max_{\mathbf{h}, \sigma^2, d(n)} p(\mathbf{X} | \mathcal{H}_1, \mathbf{h}, \sigma^2, d(n))}{\max_{\sigma^2} p(\mathbf{X} | \mathcal{H}_0, \sigma^2)} \\ &\approx \prod_{g=0}^{G-1} \frac{\max_{\mathbf{h}_g, \sigma_g^2} p(\mathbf{X}_g | \mathcal{H}_1, \mathbf{h}_g, \sigma_g^2)}{\max_{\sigma_g^2} p(\mathbf{X}_g | \mathcal{H}_0, \sigma_g^2)} \end{aligned} \quad (20)$$

where the ML estimate of σ_g^2 is given by (13) (14) using $\hat{\mathbf{R}}_{x_g} = 1/N \sum_{n=0}^{N-1} \mathbf{x}_g(n) \mathbf{x}_g^H(n)$. To derive the actual test statistics, we consider two channel models described in section II.

B.1 Phased-array model

First, for the phased-array model, it can be easily shown that

$$\mathbf{h}_g = \mathbf{h}_0 \exp(j2\pi gPD) \sin \theta, \quad g = 1, \dots, G-1 \quad (21)$$

By substituting (21) into (18) and using the approximation (19), the likelihood function (18) becomes

$$\begin{aligned} p(\mathbf{X} | \mathcal{H}_1, \mathbf{h}, \sigma^2, d(n)) &\approx \prod_{n=0}^{N-1} \prod_{g=0}^{G-1} \frac{1}{\pi^M |\mathbf{h}_0 \mathbf{h}_0^H + \sigma^2 \mathbf{I}|} \times \\ &\exp \left[-\frac{\mathbf{x}_g^H(n) (\mathbf{h}_0 \mathbf{h}_0^H + \sigma^2 \mathbf{I})^{-1} \mathbf{x}_g(n)}{\sigma^2} \right] \end{aligned} \quad (22)$$

Hence, the likelihood function only depends on $\mathbf{h}_0$. Note that the groups of spatial samples (i.e., $\mathbf{x}_0(n), \dots, \mathbf{x}_{G-1}(n)$) and time samples (e.g. $\mathbf{x}_0(0), \dots, \mathbf{x}_0(N-1)$) take the same role in (22). Thus, the above likelihood function leads to the M-GRLT test which uses P antennas. Test statistics (20) can then be written just as (15) by treating the extra spatial samples as time samples. For example, for fast sensing with $N = 1$, we may simply stack the groups of spatial samples to form the covariance matrix, that is, $\hat{\mathbf{R}}_x = \frac{1}{G} \mathbf{X} \mathbf{X}^H$ where $\mathbf{X} = [\mathbf{x}_0(0), \mathbf{x}_2(0), \dots, \mathbf{x}_{G-1}(0)]$.

B.2 Rayleigh fading model

Next, for the Rayleigh fading model, using the approximation (19) and the properties of the channel coefficients in (18), the proposed test statistic (20) can now be expressed as ,

$$T_{\text{proposed}} \propto \frac{1}{\prod_{g=0}^{G-1} \frac{\hat{\lambda}_{1,g}}{\sum_{m=1}^P \hat{\lambda}_{m,g}} \left[\prod_{g=0}^{G-1} \left(1 - \frac{\hat{\lambda}_{1,g}}{\sum_{m=1}^P \hat{\lambda}_{m,g}} \right) \right]^{P-1}} \quad (23)$$

where $\hat{\lambda}_{1,g}, \dots, \hat{\lambda}_{P,g}$ denote the eigenvalues, arranged in ascending order, of the g -th group covariance matrix given as $\hat{\mathbf{R}}_{x_g} = 1/N \sum_{n=0}^{N-1} \mathbf{x}_g(n) \mathbf{x}_g^H(n)$ whose dimension is $P \times P$. The derivation of the above test statistics is detailed in Appendix I. Notice that it only requires the calculation of P eigenvalues as opposed to M eigenvalues in the M-GRLT method.

Complexity comparison: The complexity of the M-GRLT method can be estimated as follows. It requires M^2N multiplications and $M^2(N-1)$ additions for the calculation of the covariance matrix, $\hat{\mathbf{R}}_x$, and $\mathcal{O}(M^3)$ (multiplications and additions) for the calculation of eigenvalues. For the proposed method, it requires MPN multiplications and $MP(N-1)$ additions for the calculation of all G covariance matrices. Also it requires $\mathcal{O}(MP^2)$ for the calculation of all eigenvalues. It is clear that the complexity of the proposed method only grows linearly with M whereas the M-GRLT grows at M^3 . Thus, the proposed method is suitable even when M is large while the complexity of the M-GRLT method can no longer be afforded.

IV. SIMULATION RESULTS AND DISCUSSION

The simulation consists of two parts, (1) the phased-array model, and (2) the Rayleigh fading channel model. The threshold for a given test statistic is determined by 10000 Monte Carlo simulations, with target probability of false alarm equal to 10%.

A. Phased-array model

Figure 1 shows the probability of detection against the SNR for the proposed method with the group-size, P , equal to five and for different values of M . As expected, the performance improves when M increases, and thanks to the fixed P , the complexity does not grow rapidly for the proposed method. Figure 1 also shows the M-GLRT method with $M = 5$ and $N = 20$. The reason for choosing $M = 5$ in the M-GLRT because it now needs to compute the covariance matrix of the

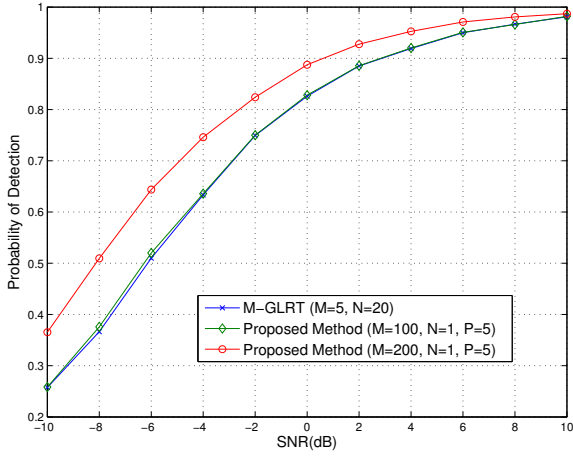


Fig. 1: Probability of detection when $P = 5$ (phased-array)

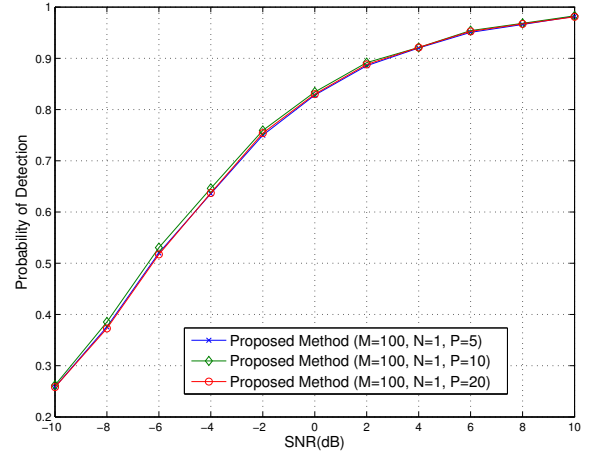


Fig. 2: Probability of detection when $P = 5, 10, 20$ (phased-array)

same dimension as that in the proposed method with $P = 5$. Note that the performance of M-GLRT is almost identical to the proposed method with $M = 100$ and $N = 1$, and in that case they both have equal number of received signal samples, combining those from the time and spatial domain. Thus, with comparable level of complexity, the proposed method reduces significantly the required sensing time by exploiting the large receive antenna array. And this proves that our method may leverage on the extra spatial samples as though they are the time domain samples to improve the spectrum sensing performance. Figure 2 then shows the results for different values P . It is clear that the performance is nearly the same as P varies. So for the phased-array model, the group size may not significantly impact the performance.

B. Rayleigh channel model

Figure 3 shows the results of the proposed method for different number of M with $P = 5$ and $N = 2$. It is evident that the proposed method may improve the spectrum sensing performance when M increases while a small N is sufficient, thus reducing the time required for accurate spectrum sensing. However, compared with the phased-array model, M has to be much larger ($M > 400$) in order to provide comparable performance as the M-GLRT method with $M = 5$ and $N = 40$, whose complexity is similar to the proposed method with $P = 5$. Figure 4 shows the probability of detection for the proposed method with $M = 100$, $N = 2$ and different values of P . Unlike the phased-array model, the performance improves significantly as P increases. Thus, the proposed method may generally need a higher P in order to deliver good performance for the Rayleigh channel model, at the expense of higher complexity.

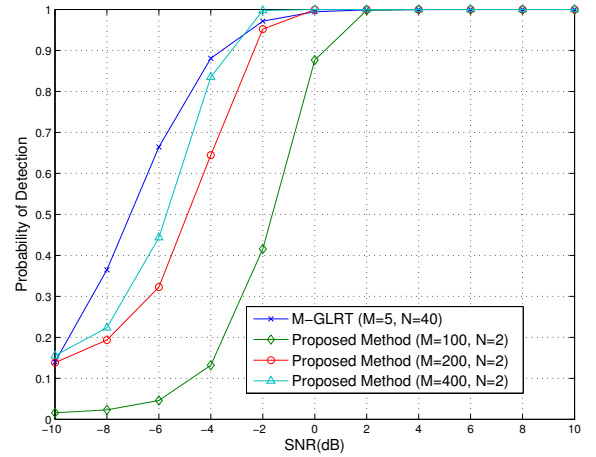


Fig. 3: Probability of detection when $P = 5$ (Rayleigh fading)

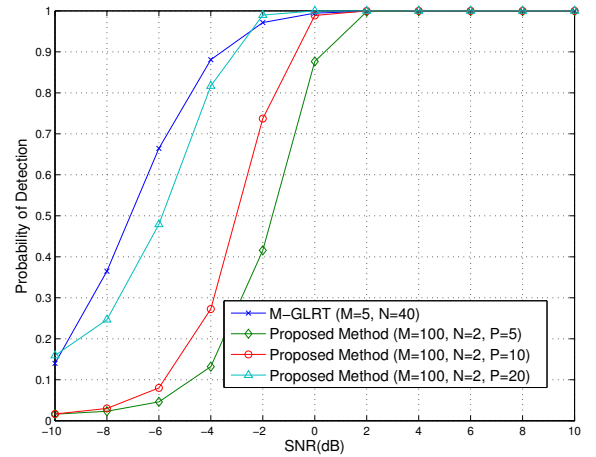


Fig. 4: Probability of detection when $P = 5, 10, 20$ (Rayleigh fading)

V. CONCLUSION

In this paper, a novel GLRT-based detector has been proposed for spectrum sensing with large number of receive antennas. As previous GLRT-based approaches which involve the computation of eigenvalues of covariance matrix may

require too much computational burden as the number of antenna M increases, the proposed method simplifies the test statistics by decomposing the detection based on small groups of received spatial samples. Like the additional time domain

signal samples, the excessive spatial samples can be utilized in our method to improve the spectrum sensing performance while the complexity remains at an reasonable level. In order words, the advantage of large number of receive antennas translates into saving of time domain samples in the spectrum sensing process. The simulations results show that the larger the M , the better is the probability of detection using our method while the complexity remains practical. Thus, the proposed method is a fast and effective spectrum sensing method leveraging the large number of receive antennas to improve the spectrum sensing performance.

VI. APPENDIX

DERIVATION FOR THE PROPOSED TEST STATISTIC

The derivation resembles the one in [7] with some modifications. Similar to (12)-(14), we can form ML estimate of $\mathbf{h}_g$, $\sigma_{\mathcal{H}_0}^2$, $\sigma_{\mathcal{H}_1}^2$ using the g -th group's covariance matrix $\mathbf{R}_{x_g}$. By substituting these ML estimates into the log-likelihood functions (17) (18) and using the approximation (19), we obtain the following:

$$\begin{aligned} \ln p(\mathbf{X}|\mathcal{H}_0, \hat{\sigma}_{\mathcal{H}_0}^2) &\approx \\ \sum_{g=0}^{G-1} \left\{ -\frac{PN}{2} [\ln(2\pi) + 1] - \frac{PN}{2} \ln \left(\frac{1}{P} \sum_{m=1}^P \hat{\lambda}_{m,g} \right) \right\} \\ \ln p(\mathbf{X}|\mathcal{H}_1, \hat{\mathbf{h}}, \hat{\sigma}_{\mathcal{H}_1}^2) &\approx \\ \sum_{g=0}^{G-1} \left\{ -\frac{PN}{2} [\ln(2\pi) + 1] \right. \\ &\quad \left. - \frac{(P-1)N}{2} \ln \left(\frac{1}{P-1} \sum_{m=2}^P \hat{\lambda}_{m,g} \right) - \frac{N}{2} \ln(\hat{\lambda}_{1,g}) \right\} \quad (24) \end{aligned}$$

Using the above functions and after ignoring some constant terms, the proposed log-statistic (9) can be expressed as,

$$\begin{aligned} T_{\log-proposed} = \\ N \sum_{g=0}^{G-1} \left\{ \ln \left[\left(\frac{\sum_{m=1}^P \hat{\lambda}_{m,g}}{\sum_{m=2}^P \hat{\lambda}_{m,g}} \right)^{P-1} \left(\frac{\sum_{m=1}^P \hat{\lambda}_{m,g}}{\hat{\lambda}_{1,g}} \right) \right] \right\} \quad (25) \end{aligned}$$

which can be further simplified to

$$T_{proposed} \propto \frac{1}{\prod_{g=0}^{G-1} \frac{\hat{\lambda}_{1,g}}{\sum_{m=1}^P \hat{\lambda}_{m,g}} \left[\prod_{g=0}^{G-1} \left(1 - \frac{\hat{\lambda}_{1,g}}{\sum_{m=1}^P \hat{\lambda}_{m,g}} \right) \right]^{P-1}} \quad (26)$$

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